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Research article

Space-time Dependence of Fine Structure Constant in Deformed Gauge Invariance

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Abstract

We consider a mechanism for the possibility of fine structure constant to be changed in space-time. It is based on the deformed gauge invariance principle proposed in [5]. The mechanism also allows the possibility for gauge bosons to acquire mass.

Keywords: Gauge theory, Fine structure constant

1. Motivation

The fine structure constant $\alpha\ (=\frac{e^2}{4\pi})$ is a fundamental parameter in electromagnetic processes. The question of whether it changes the value in space-time is a problem of significant meaning for the study of both micro- and macro world, attracting a lot of interest.

On the other hand, it is realized that if the fine structure constant is in fact a function of space-time, many phenomena of the Nature related to the time evolution of the Universe might be theoretically explained. In particular, it is worth mentioning the Red Shift in Cosmology traditionally treated as Doppler effect [1] due to the expansion of the Universe. Within the framework of our proposed mechanism it might be explained in an alternative way more compatible with static Universe in General Relativity [2,3]. Another example would be the Okolo problem [4], which also might be explained if the value of α some milliards years ago was far different from that at present time.

The aim of this paper is to propose a mechanism for space-time dependence of the fine structure constant, which is based on a version of modified gauge principle first considered in [5] and referred to as deformed gauge invariance. As a consequence the model also allows the possibility for gauge bosons to acquire mass independently of Higgs mechanism.

2. Deformed U(1) – gauge invariance

Let $\varphi(x)$ be some field having U(1) charge q and obeying the transformation law

$$\varphi(x) \to \varphi^{'}(x) = e^{-iq\omega(x)}\varphi(x) \qquad (1)$$

under gauge transformation with parameter $\omega(x)$.

The covariant derivative is introduced by the formula:

$$D_\mu\varphi(x) = \partial_\mu\varphi(x) - iqe^{g(x)}A_\mu(x)\varphi(x) \qquad (2)$$

where $A_\mu(x)$ is the gauge field with the transformation law:

$$A_\mu^{'}(x) = A_\mu(x) - e^{-g(x)}\partial_\mu\omega(x) \qquad (3)$$

$g(x)$ being some scalar function parameter with some constraint to be specified later.

The equation (1) – (3) lead to the following U(1) – gauge interaction Lagrangians:

$$L_{\text{int}}(\psi, A) = qe^{g(x)}.\overline{\psi}(x)\gamma_\mu\psi(x).A^\mu(x) \qquad (4)$$

for spinor field ψ,

$$\begin{aligned} L_{\text{int}}(\phi, A) &= qe^{g(x)}\phi^+(x)\overleftrightarrow{\partial}_\mu\phi(x).A^\mu(x) \\ &\quad + q^2e^{2g(x)}\phi^+\phi.A_\mu A^\mu \end{aligned} \qquad (5)$$

for scalar field ϕ, etc.

It follows from (4) that for electromagnetic interaction of electron instead of the fine structure constant α one should use

$$\tilde{\alpha}(x) \equiv \alpha.e^{2g(x)} \qquad (6)$$

The conventional field strength defined as

$$F_{\mu\upsilon}{}^{(0)} \equiv \partial_\mu A_\upsilon - \partial_\upsilon A_\mu \qquad (7)$$

is not invariant under the transformation (3), but its deformed version

$$F_{\mu\upsilon}{}^{(g)} \equiv F_{\mu\upsilon}{}^{(0)} + \partial_\mu g.A_\upsilon - \partial_\upsilon g.A_\mu \qquad (8)$$

Hence, the corresponding invariant free Lagrangian for gauge field

$$\begin{aligned} L_0{}^{(g)}(A) &= -\frac{1}{4}F_{\mu\upsilon}{}^{(g)}F^{(g)\mu\upsilon} \\ &= -\frac{1}{4}F_{\mu\upsilon}{}^{(0)}F^{(0)\mu\upsilon} - \frac{1}{2}(\partial_\mu g.\partial^\mu g).(A_\upsilon A^\upsilon) \\ &\quad + \frac{1}{2}(\partial_\mu g.A^\mu)^2 - F_{\mu\upsilon}{}^{(0)}.\partial^\mu g.A^\upsilon \end{aligned} \qquad (9)$$

should be used.

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Applied to eq.(9), the Euler – Lagrange equation gives:

$$\{(\Box - \partial_\upsilon g.\partial^\upsilon g + \Box g)A^\mu + \partial^\mu g.\partial_\upsilon g.A^\upsilon$$

$$- \partial^\upsilon A_\upsilon.\partial^\mu g - \partial^\mu\partial^\upsilon A_\upsilon + \partial_\upsilon g.\partial^\mu A^\upsilon - \partial^\mu\partial^\upsilon g.A_\upsilon\}=0 \qquad (10)$$

Now let us put the deformed Lorentz gauge condition on the gauge field A_μ to be:

$$\partial^\mu A_\mu = \partial^\mu g.A_\mu \qquad (11)$$

This coincides with the ordinary Lorentz gauge condition when g is constant.

With the condition (11) equation (10) becomes:

$$(\Box - \partial_\upsilon g.\partial^\upsilon g + \Box g)A^\mu - 2\partial^\mu\partial^\upsilon g.A_\upsilon = 0 \qquad (12)$$

Now we restrict the consideration to a special form of $g(x)$, namely

$$g(x) = \frac{1}{2}\gamma x^2 + p_\mu x^\mu \quad (13)$$

where γ is some constant parameter, p_μ is some vector parameter.

With the expression (13) inserted eq.(6) and eq.(12) become:

$$\tilde{\alpha}(x) = \alpha.e^{\gamma x^2 + 2px} \qquad (14)$$

$$\{\Box + 2\gamma - (\gamma x^2 + 2\gamma px + p^2\}A_\mu = 0 \qquad (15)$$

Eq. (15) corresponds to the expression

$$m_A^2 = 2\gamma - (\gamma x^2 + 2\gamma px + p^2) \qquad (16)$$

for gauge boson mass.

It follows that in general $\tilde{\alpha}$ and m_A can have different values at different space-time coordinates. In the special case $\gamma = 0$ we have:

$$\tilde{\alpha}(x) = \alpha.e^{2px} \qquad (17)$$

$$m_A^2 = -p^2$$

In particular this gives $m_A = 0$ only when $p_0^2 = \vec{p}^2$.

Alternatively, in the special case $p_\mu = 0$ instead of eq.(17) we have:

$$\tilde{\alpha}(x) = \alpha.e^{\gamma x^2} \qquad (18)$$

$$m_A^2 = (2 - x^2)\gamma$$

In this case the fine structure constant remains to be unchanged only on the light cone $x^2 = 0$, and A_μ remains to be massless only on the 4-sphere $x^2 = 2$.

3. Deformed non-abelian gauge

The results obtained above can be straightly generalized for the case of non-abelian gauge.

Let $\varphi_i(x)$ be some field multiplet with the transformation law

$$\varphi_i'(x) = \{S(x)\}_i^j \varphi_j(x) \qquad (19)$$

$$S(x) = e^{-i\sum_a \omega_a(x) M_a}$$

M_a being representation matrices.

The covariant derivative is introduced by the formula:

$$D_\mu \varphi_i(x) = \partial_\mu \varphi_i(x) - iqe^{g(x)} (A_\mu)_i^j \varphi_j \qquad (20)$$

$$A_\mu \equiv \sum_a A_{\mu a} M_a$$

with the transformation law

$$A_\mu'(x) = S A_\mu S^{-1} + \frac{i}{q} e^{-g(x)} S \partial_\mu S^{-1} \qquad (21)$$

The deformed field strength is defined in a similar way as eq.(8), namely:

$$\begin{aligned} F_{\mu\upsilon a}^{(g)} &= \partial_\mu A_{\upsilon a} - \partial_\upsilon A_{\mu a} + qe^{g(x)} \sum_{b,c} f_{abc} A_{\mu b} A_{\upsilon c} \\ &+ \partial_\mu g . A_{\upsilon a} - \partial_\upsilon g . A_{\mu a} \end{aligned} \qquad (22)$$

f_{abc} being structure constants of the gauge group.

It transforms according to the formula:

$$F_{\mu\upsilon a}^{(g)'} = S F_{\mu\upsilon a}^{(g)} S^{-1} \qquad (23)$$

$$F_{\mu\upsilon}^{(g)} \equiv \sum_a F_{\mu\upsilon a}^{(g)} M_a$$

Hence, the invariant Lagrangian for the gauge fields is:

$$\begin{aligned} L^{(g)}(A) &= -\frac{1}{2} Tr F_{\mu\upsilon}^{(g)} F^{(g)\mu\upsilon} \\ &= L_0^{(g)}(A) + L_{\text{int}}^{(g)}(A) \end{aligned} \qquad (24)$$

where

$$\begin{aligned} L_0^{(g)}(A) &\equiv -\frac{1}{4} F_{\mu\upsilon a}^{(0)} F_a^{(0)\mu\upsilon} - \frac{1}{2} (\partial_\mu g . \partial^\mu g)(A_{\upsilon a} A_a^\upsilon) \\ &+ \frac{1}{2} (\partial_\mu g . A_a^\mu)^2 - F_{\mu\upsilon}^{(0)} . \partial^\mu g . A_a^\upsilon \end{aligned} \qquad (25)$$

is the free Lagrangian for gauge fields,

$$F_{\mu\upsilon a}^{(0)} \equiv \partial_\mu A_{\upsilon a} - \partial_\upsilon A_{\mu a}$$

$L_{\text{int}}^{(g)}(A)$ is the interaction Lagrangian between gauge fields, which is proportional to $qe^{g(x)}$ and $\left(qe^{g(x)}\right)^2$.

The Euler – Lagrangian equation applied to the Lagrangian (24) gives:

$$\left\{ \Box - \partial_\upsilon g . \partial^\upsilon g + \Box g \right\} A_{\mu a} = (\partial_\mu + \partial_\mu g)(\partial^\upsilon A_{\upsilon a} - \partial^\upsilon g . A_{\upsilon a}) + 2 \partial_\mu \partial_\upsilon g . A_a^\upsilon + qe^{g(x)} \{ \ldots \} \qquad (26)$$

With the same restriction (13) on the function parameter (13) and the deformed Lorentz gauge condition

$$\partial^\mu A_{\mu a} = \partial^\mu g . A_{\mu a} \qquad (27)$$

The equation (26) becomes:

$$(\Box + m_A^2) A_{\mu a} = q e^{g(x)} \{\ldots\} \qquad (28)$$

With the mass m_A defined by the same formula (16).

4. Conclusion

In this work we have considered a mechanism for space – time dependence of the gauge coupling constants. It is based on the concept of deformed gauge invariance by introducing some parameter function $g(x)$ in the gauge transformation. The ordinary gauge invariance corresponds to the case $g(x) = 0$. The mechanism allows also the possibility for the gauge bosons to acquire mass.

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